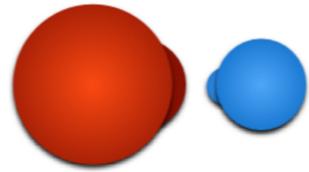


# Interpolation of Population Synthesis Simulations for Compact Binary Mergers

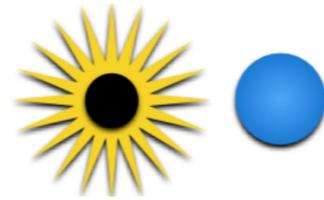
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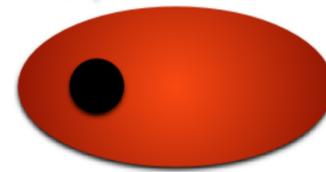
1. Take two massive stars. If they were on their own they would evolve along the main sequence.



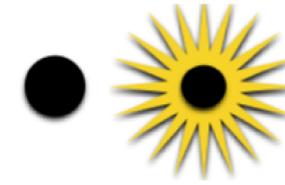
2. As one of the stars expands, the mutual gravitational pull starts to deform both stars. Some mass may transfer between them.



3. One of the stars runs out of fuel and goes supernova, forming a black hole or a neutron star.



4. As the remaining star continues to expand, its outer layers engulf the compact object into a 'common envelope'.



5. The second star goes supernova, forming another black hole or neutron star.



6. The binary orbit shrinks as energy is lost by gravitational radiation, and the stars eventually merge into a single compact object.

## Gravitational Waves and Population Synthesis

Since September the Laser Interferometer Gravitational-Wave Observatory (LIGO) has confidently detected two black hole mergers by measuring gravitational waves here on earth (Abbott et al. 2016; PRL 116, 061102). These events tell us a great deal about the final moments of these binaries' lifetimes. However, we also want to learn about how these binaries formed in the first place.

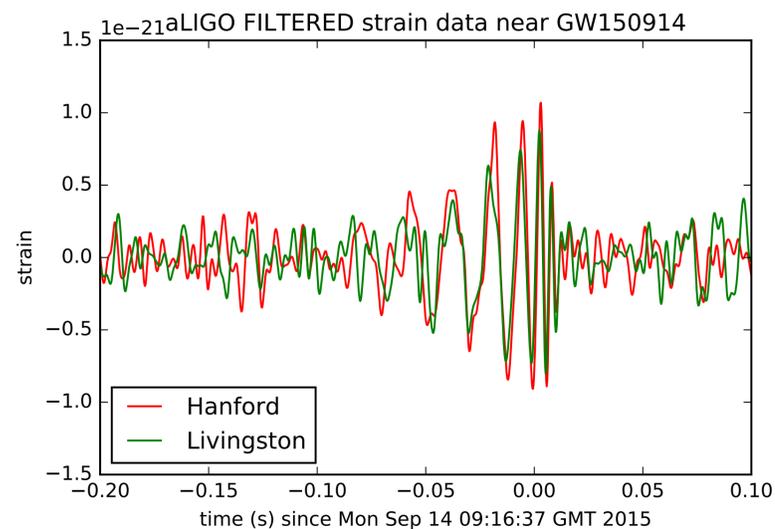


Figure: LIGO observations of gravitational wave GW150914, plotted using public data from <https://lsc.ligo.org>

Population Synthesis simulations calculate the evolution of many isolated binaries, starting from a pair of normal main sequence stars. The simulations are governed by a number of 'hyperparameters' which control parts of the evolution which are theoretically uncertain.

For example, when a star goes supernova, it is theorised that asymmetries can give the star a 'kick'. The strength of such kicks is included as a hyperparameter.

## The Inference Problem

It is expected that over the next few years LIGO will observe many tens of compact binary mergers. The ultimate goal will be to use LIGO observations to infer posteriors on the population synthesis hyperparameters.

However, not all binaries form LIGO sources, and the lifetime of a binary is not deterministic. It is often necessary to simulate many millions systems to get a few thousand LIGO sources for a given set of hyperparameters. Moreover, the dimensionality of the space of hyperparameters is relatively high ( $\sim 10$  dimensions), with unknown correlations and degeneracies. This combination of factors quickly renders conventional inference computationally intractable.

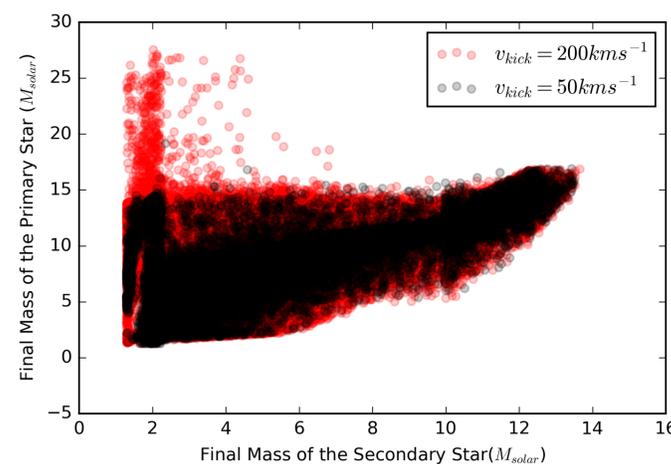


Figure: The final mass plane for two choices of (nearby) hyperparameters, in which only the strength of supernova kicks has been changed. Each point represents a single binary. These populations were generated using the BSE code (Hurley et al. 2002; arXiv:astro-ph/0201220)

Therefore, the problem we are trying to solve is how to explore the large space of hyperparameters with a limited number of actual binary simulations available to us.

## Interpolation of Distributions

The problem of exploring the regions of the hyperparameter space between the population synthesis simulations is made significantly more difficult by the stochastic nature of binary evolution. The same set of initial conditions for the same pair of main sequence stars does not necessarily result in the same final properties of the system.

Therefore the physical quantities of interest (e.g final masses of the compact objects) exist as distributions at each point in hyperparameter space. We are still actively researching the best method for interpolating between these distributions. The most promising approach so far is based on a nearest neighbours regression, and is closely related to the approximate Bayesian computation (ABC) method.

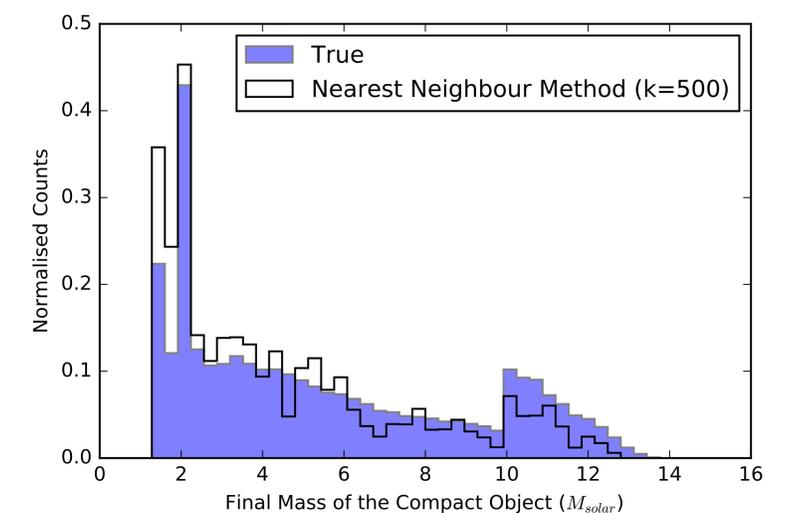


Figure: Plots of the final mass of one of the compact objects in a binary. In blue is the distribution of masses for a fixed set of hyperparameters. In black is the prediction of the distribution of masses at the fixed hyperparameters, given a dataset where 3 of the hyperparameters are allowed to vary.